SURFACE AREA AND VOLUME OF SOLIDS

Geometry

Chapter 12

- This Slideshow was developed to accompany the textbook
  - *Larson Geometry*
  - *By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.*
  - *2011 Holt McDougal*
- Some examples and diagrams are taken from the textbook.

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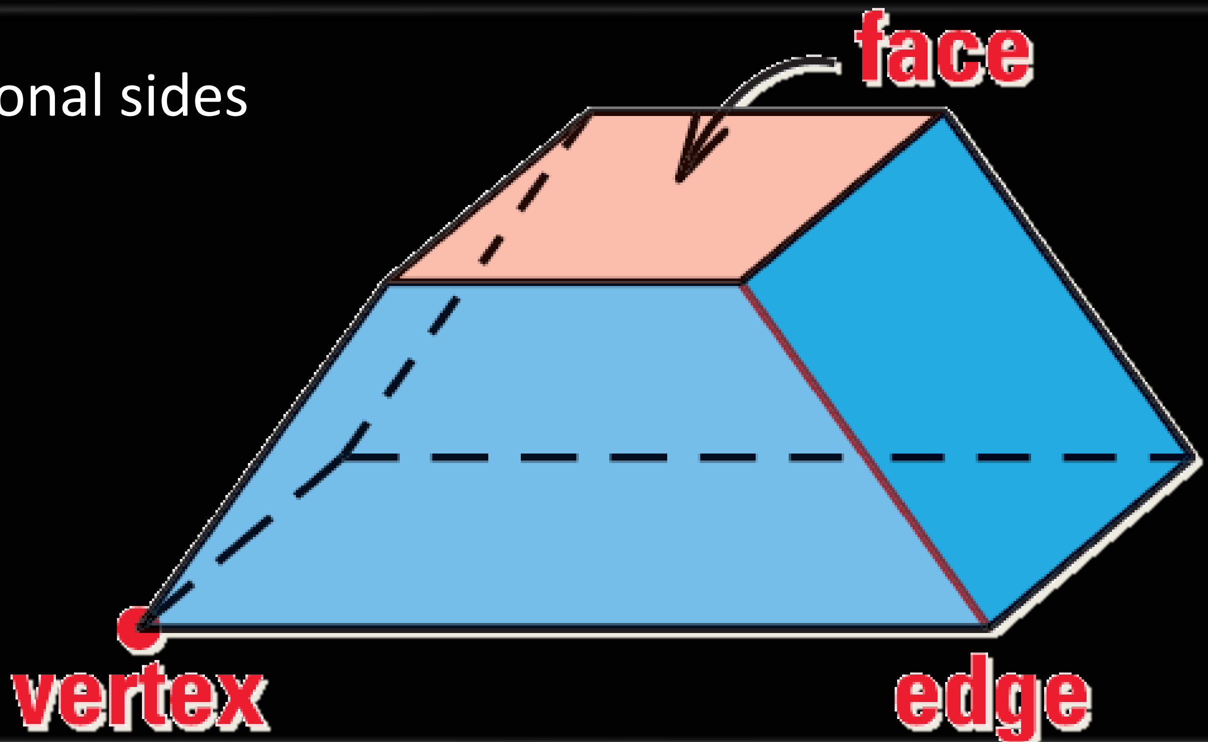
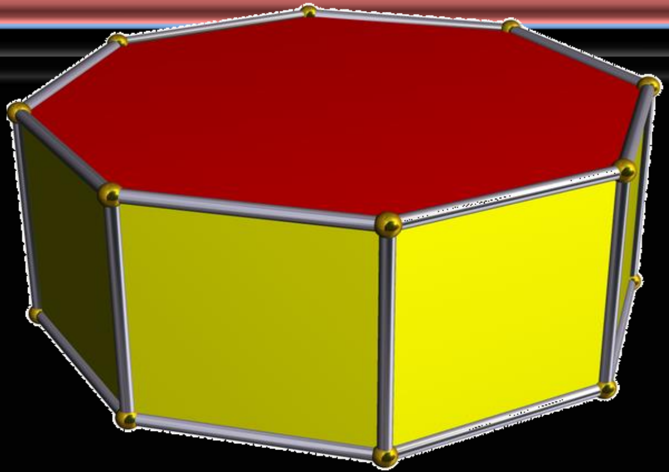
12.1 EXPLORE SOLIDS

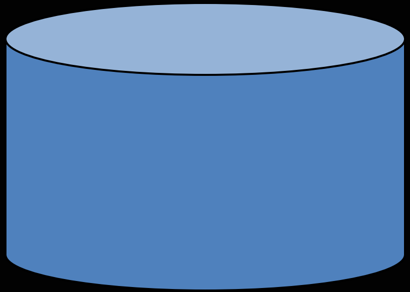
- Polyhedron
  - Solid with polygonal sides
  - Flat sides
- Face
  - Side
- Edge
  - Line segment
- Vertex
  - Corner



12.1 EXPLORE SOLIDS

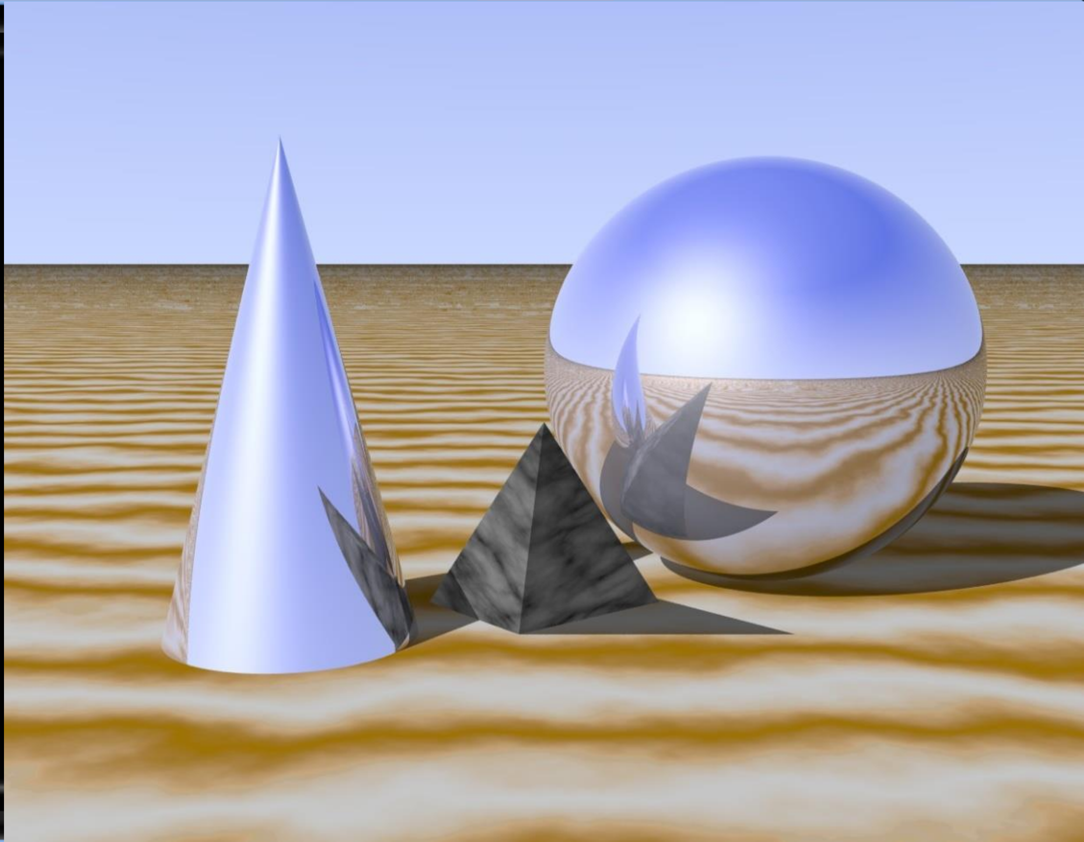
- Prism
  - Polyhedron with two congruent surfaces on parallel planes (the 2 ends (**bases**) are the same)
  - Named by bases (i.e. rectangular prism, triangular prism)
- Cylinder
  - Solid with congruent circular bases on parallel planes (not a polyhedron)





12.1 EXPLORE SOLIDS

- Pyramid → polyhedron with all but one face intersecting in one point
- Cone → circular base with the other surface meeting in a point (kind of like a pyramid)
- Sphere → all the points that are a given distance from the center



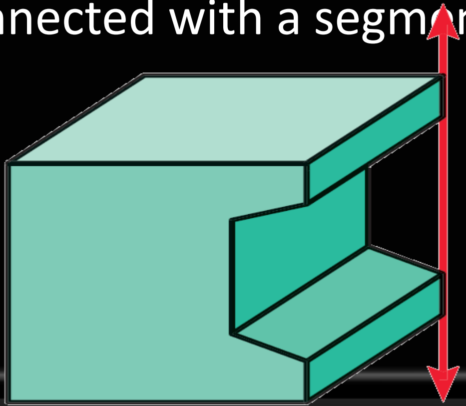
12.1 EXPLORE SOLIDS

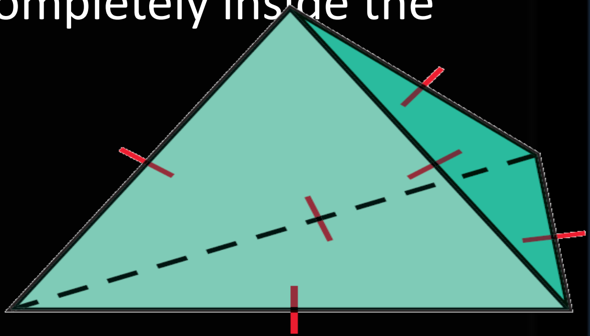
Euler's Theorem

The number of faces (*F*), vertices (*V*), and edges (*E*) of a polyhedron are related by

$$F + V = E + 2$$

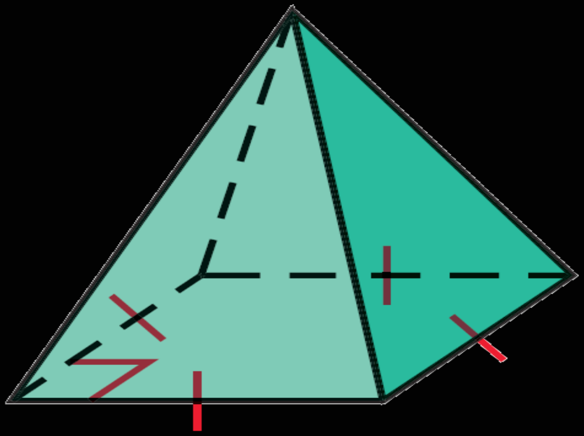
- Convex
  - Any two points can be connected with a segment completely inside the polyhedron
- Concave
  - Not convex
  - Has a "cave"



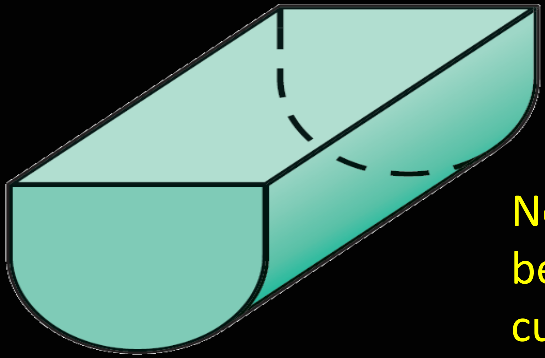


12.1 EXPLORE SOLIDS

- Tell whether the solid is a polyhedron. If it is, name the polyhedron and find the number of faces, vertices, and edges and describe as convex or concave



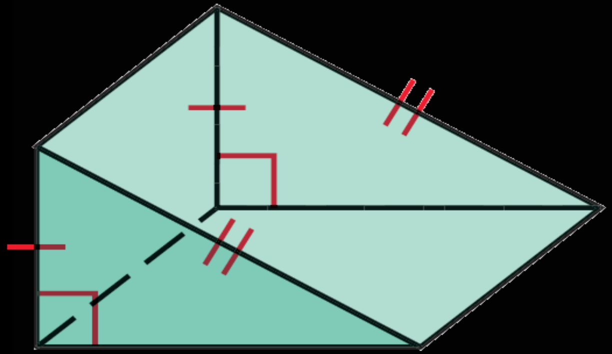
Polyhedron
Square Pyramid
5 faces
5 vertices
8 edges
convex



Not a polyhedron because of the curved face

12.1 EXPLORE SOLIDS

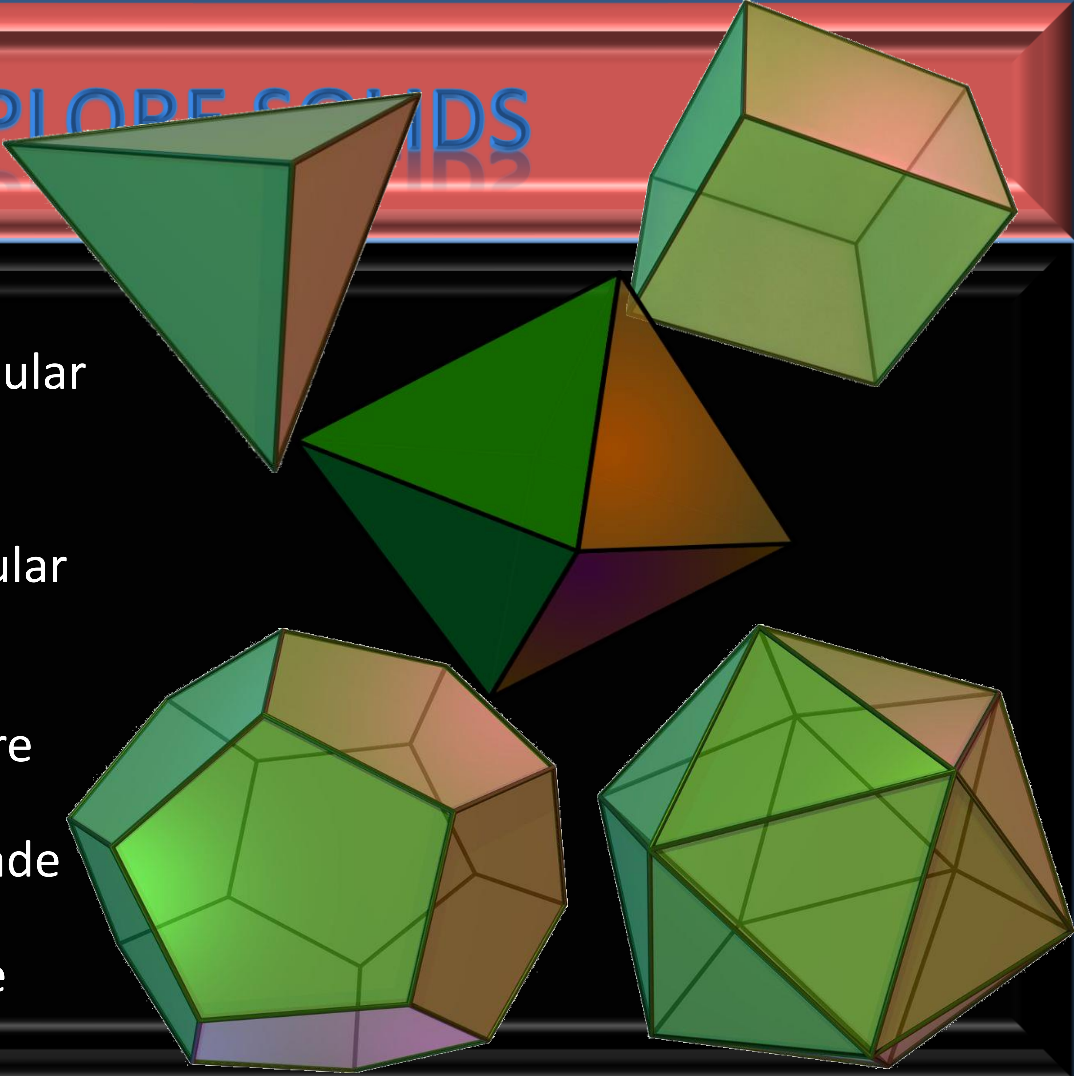
- Tell whether the solid is a polyhedron. If it is, name the polyhedron and find the number of faces, vertices, and edges and describe as convex or concave



Polyhedron
Triangular Prism
5 faces
6 vertices
9 edges
convex

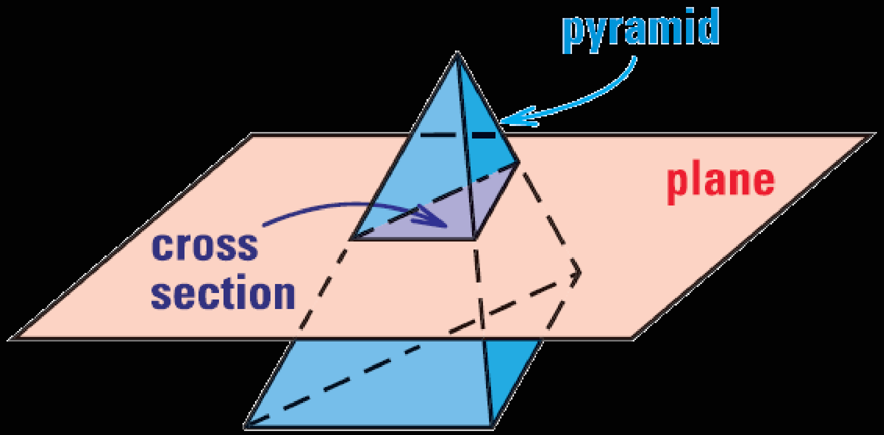
12.1 EXPLORE SOLIDS

- Regular polyhedron
  - Polyhedron with congruent regular polygonal faces
- Only 5 types (**Platonic solids**)
  - **Tetrahedron** → 4 faces (triangular pyramid)
  - **Hexahedron** → 6 faces (cube)
  - **Octahedron** → 8 faces (2 square pyramids put together)
  - **Dodecahedron** → 12 faces (made with pentagons)
  - **Icosahedron** → 20 faces (made with triangles)



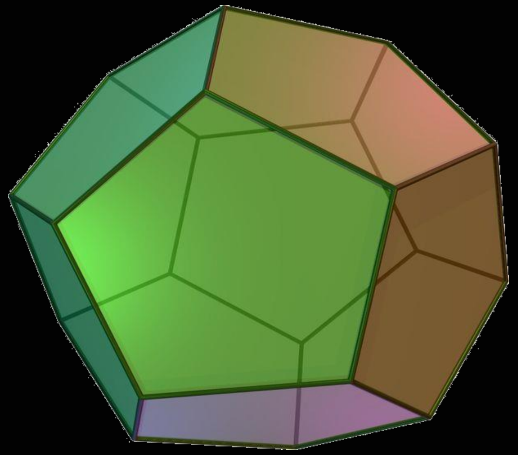
12.1 EXPLORE SOLIDS

- Cross Section
  - ◇ Imagine slicing a very thin slice of the solid
  - ◇ The cross section is the 2-D shape of the thin slice



12.1 EXPLORE SOLIDS

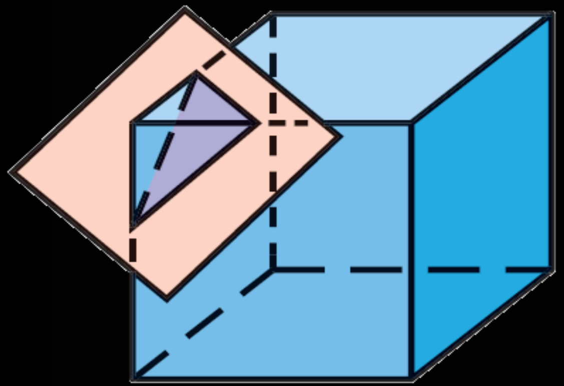
- Find the number of faces, vertices, and edges of a regular dodecahedron. Check with Euler's Theorem.



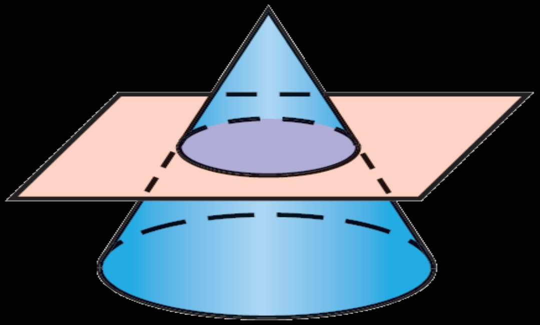
- 12 Faces
- 20 Vertices
- 30 Edges
- $F + V = E + 2$
- $12 + 20 = 30 + 2$
- $32 = 32$

12.1 EXPLORE SOLIDS

- Describe the cross section.



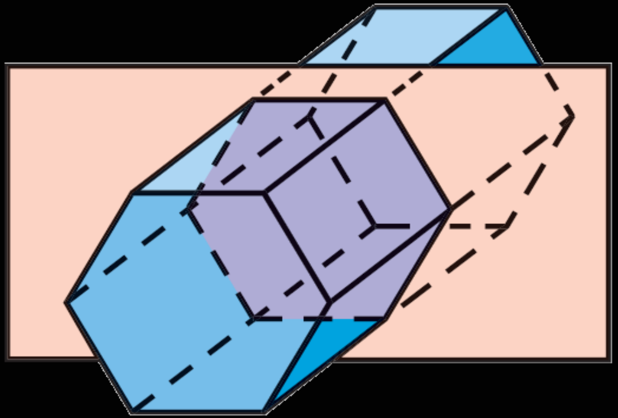
Triangle



Circle

12.1 EXPLORE SOLIDS

- Describe the cross section.



hexagon

- *798 #2-40 even, 44-60 even = 29*

ANSWERS AND QUIZ

- 12.1 Answers
- 12.1 Homework Quiz

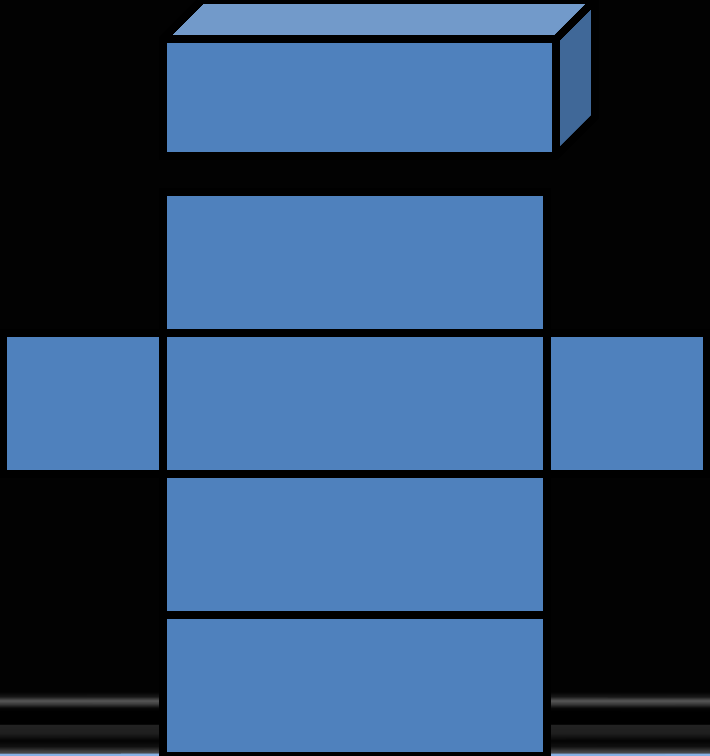
12.2 SURFACE AREA OF PRISMS AND CYLINDERS

- Surface area = sum of the areas of each surface of the solid
    - In order to calculate surface area it is sometimes easier to draw all the surfaces

12.2 SURFACE AREA OF PRISMS AND CYLINDERS

Nets

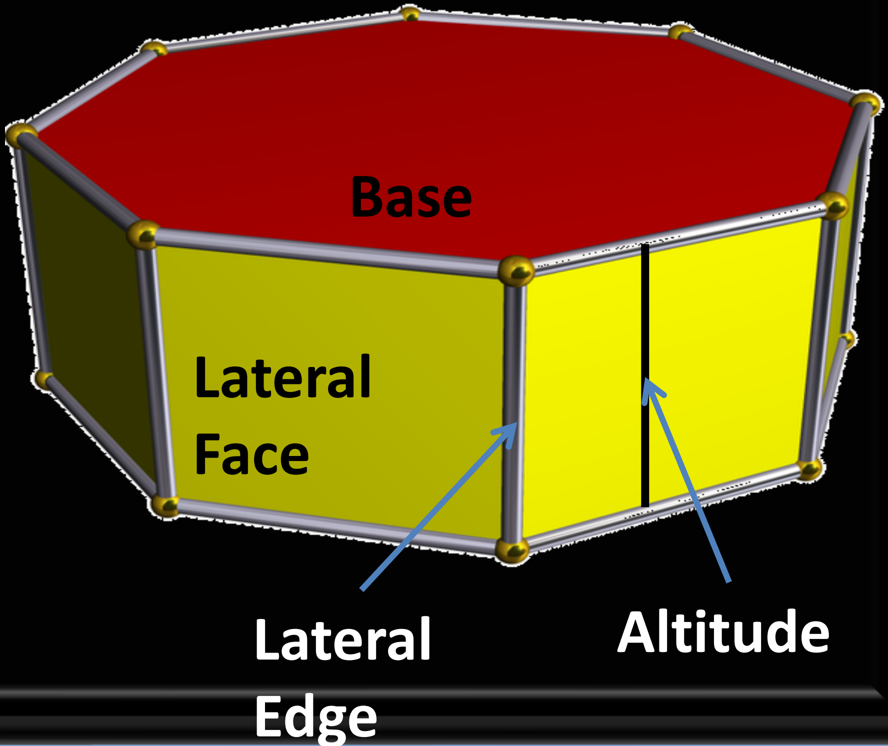
- Imagine cutting the three dimensional figure along the edges and folding it out.
- Start by drawing one surface, then visualize unfolding the solid.
- To find the surface area, add up the area of each of the surfaces of the net.



12.2 SURFACE AREA OF PRISMS AND CYLINDERS

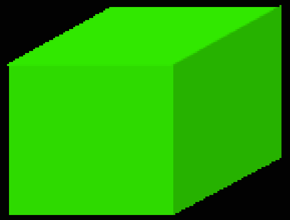
Parts of a right prism

- Bases → parallel congruent surfaces (the ends)
- Lateral faces → the other faces (they are parallelograms)
- Lateral edges → intersections of the lateral faces (they are parallel)
- Altitude → segment perpendicular planes containing the two bases with an endpoint on each plane
- Height → length of the altitude

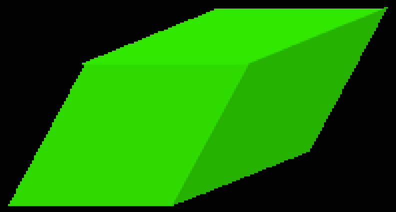


12.2 SURFACE AREA OF PRISMS AND CYLINDERS

- Right prism
  - Prism where the lateral edges are altitudes



- Oblique prism
  - Prism that isn't a right prism



12.2 SURFACE AREA OF PRISMS AND CYLINDERS

Lateral Area (L) of Prisms

- Area of the Lateral Faces

- $L = Ph$
  - L = Lateral Area
  - P = Perimeter of base
  - h = Height

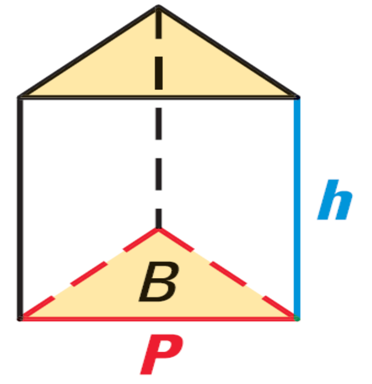
12.2 SURFACE AREA OF PRISMS AND CYLINDERS

- Base Area (B)
  - In a prism, both bases are congruent, so you only need to find the area of one base and multiply by two

Surface Area of a Right Prism

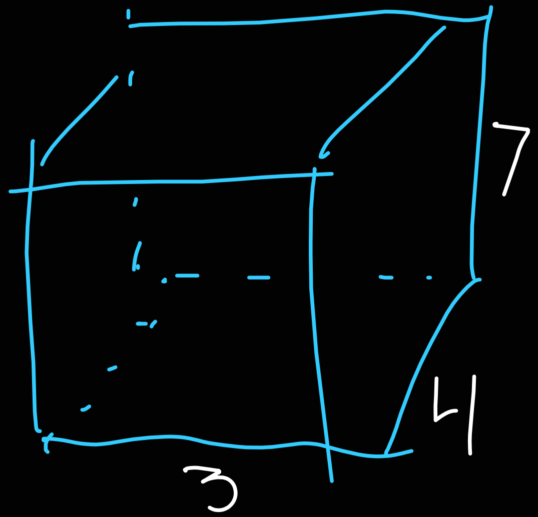
$$S = 2B + Ph$$

Where S = surface area, B = base area, P = perimeter of base, h = height of prism



12.2 SURFACE AREA OF PRISMS AND CYLINDERS

- Draw a net for a triangular prism.



- Find the lateral area and surface area of a right rectangular prism with height 7 inches, length 3 inches, and width 4 inches.
- $P = 2\ell + 2w$
- $P = 2(3) + 2(4) = 14$
- $L = Ph = (14)(7) = 98$
- $B = bh = 3 \cdot 4 = 12$
- $A = 2B + Ph$
- $A = 2(12) + 14(7) = 122$

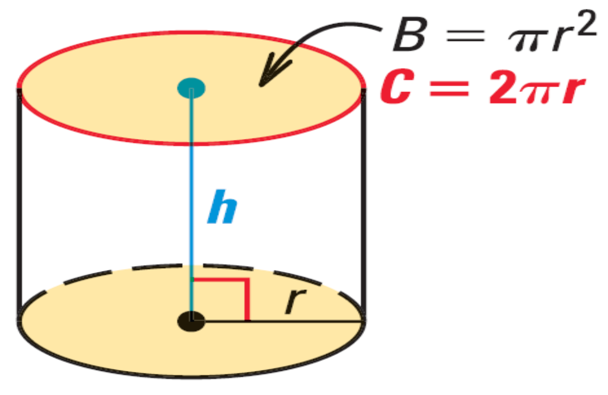
12.2 SURFACE AREA OF PRISMS AND CYLINDERS

- Cylinders are the same as prisms except the bases are circles
  - Lateral Area = L = 2πrh

Surface Area of a Right Cylinder

$$S = 2\pi r^2 + 2\pi rh$$

Where S = surface area, r = radius of base, h = height of prism

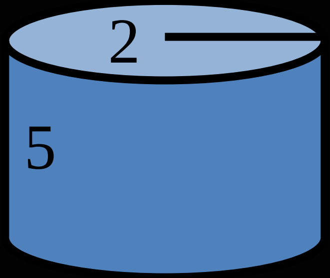


12.2 SURFACE AREA OF PRISMS AND CYLINDERS

- The surface area of a right cylinder is 100 cm^2. If the height is 5 cm, find the radius of the base.
- $100 = 2\pi r^2 + 2\pi r(5)$
- $100 = 2\pi r^2 + 10\pi r$
- $0 = 2\pi r^2 + 10\pi r - 100$
- $0 = r^2 + 5r - 15.915$
- $r = \frac{-5 \pm \sqrt{5^2 - 4(1)(-15.915)}}{2(1)}$
- $r = \frac{-5 \pm \sqrt{88.662}}{2}$
- $r = 2.2, -7.2$
- Only 2.2 makes sense because the radius must be positive

12.2 SURFACE AREA OF PRISMS AND CYLINDERS

- Find of the cylinder surface area.



- *806 #2-28 even, 31-37 all = 21*

- $S = 2\pi r^2 + 2\pi rh$
- $S = 2\pi 2^2 + 2\pi(2)(5)$
- $S = 8\pi + 20\pi$
- $S = 28\pi$

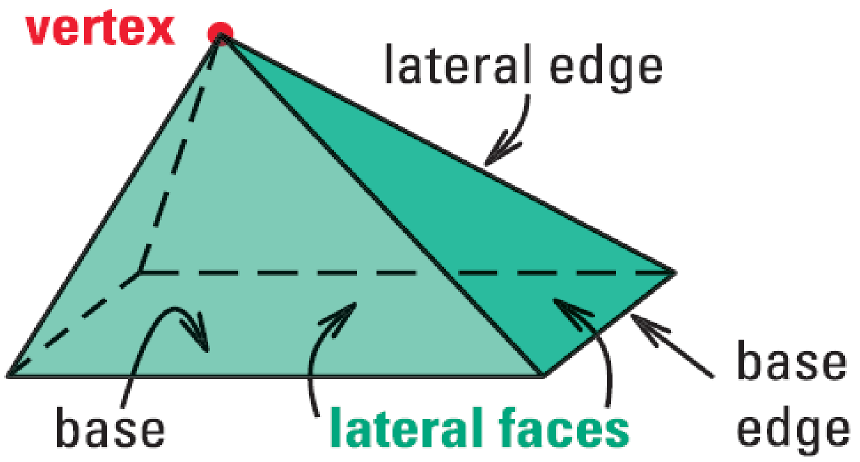
ANSWERS AND QUIZ

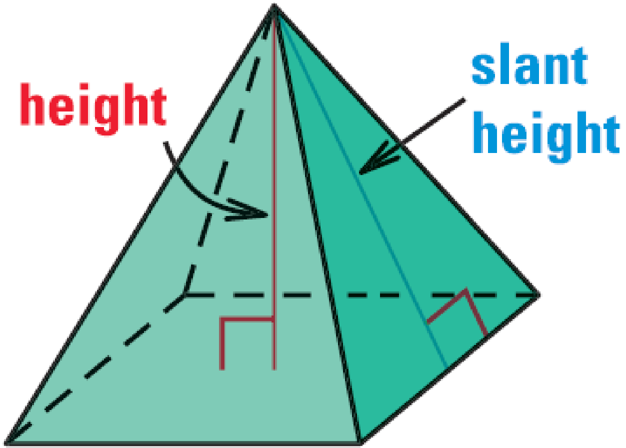
- 12.2 Answers
- 12.2 Homework Quiz

12.3 SURFACE AREA OF PYRAMIDS AND CONES

Pyramids

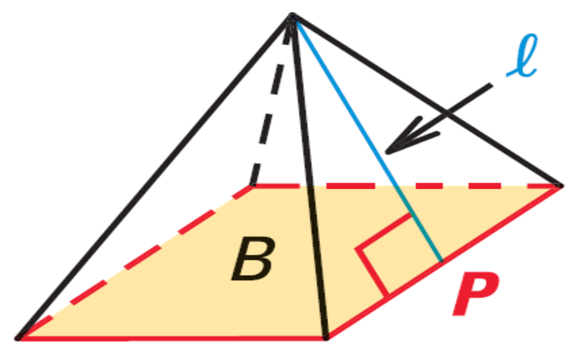
- All faces except one intersect at one point called **vertex**
- The **base** is the face that does not intersect at the vertex
- **Lateral faces** → faces that meet in the vertex
- **Lateral edges** → edges that meet in the vertex
- **Altitude** → segment that goes from the vertex and is perpendicular to the base





12.3 SURFACE AREA OF PYRAMIDS AND CONES

- **Regular pyramid** → base is a regular polygon and the vertex is directly above the center of the base
  - In a regular pyramid, all the lateral faces are congruent isosceles triangles
  - The height of each lateral face is called the **slant height** (ℓ)
- **Lateral Area** → L = ½ Pℓ



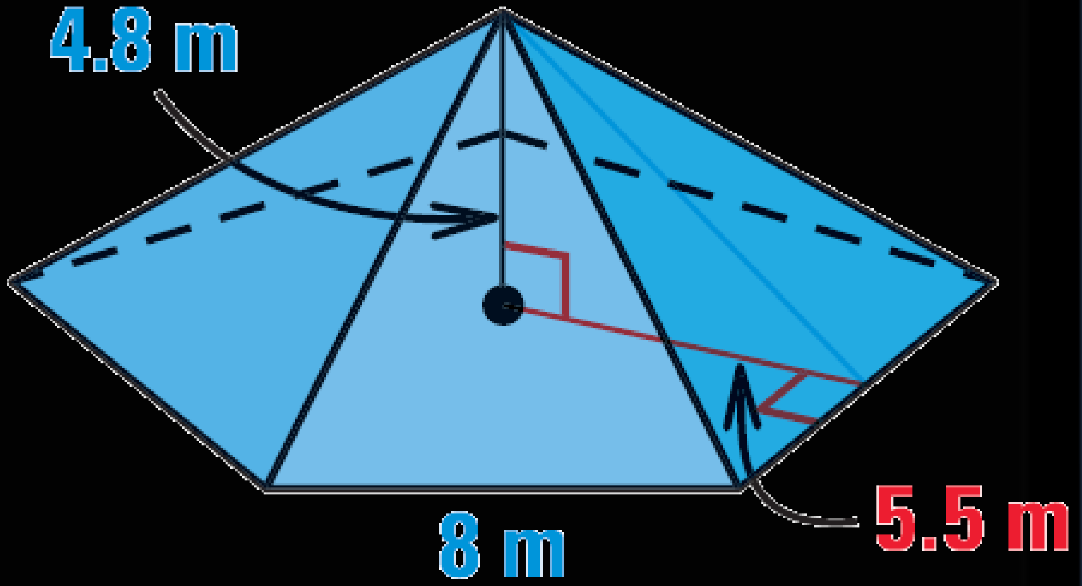
Surface Area of a Regular Pyramid

$$S = B + \frac{1}{2}P\ell$$

Where B = base area, P = base perimeter, ℓ = slant height

12.3 SURFACE AREA OF PYRAMIDS AND CONES

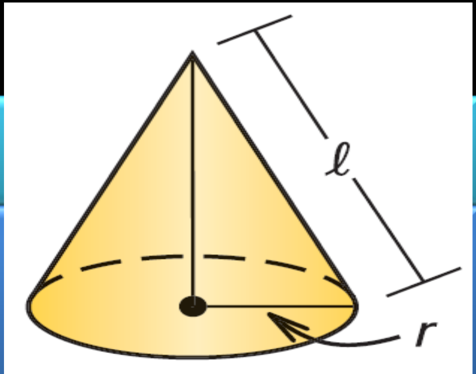
- Find the surface area of the regular pentagonal pyramid.
- $B = \frac{1}{2}Pa$
- $B = \frac{1}{2}(5 \cdot 8)(5.5) = 110$
- $\ell^2 = 5.5^2 + 4.8^2$
- $\ell = 7.3$
- $S = B + \frac{1}{2}P\ell$
- $S = 110 + \frac{1}{2}(5 \cdot 8)(7.3) = 256$



12.3 SURFACE AREA OF PYRAMIDS AND CONES

Cones

- Cones are just like pyramids except the base is a circle
- Lateral Area = $\pi r \ell$



Surface Area of a Right Cone

$$S = \pi r^2 + \pi r \ell$$

Where r = base radius, ℓ = slant height

12.3 SURFACE AREA OF PYRAMIDS AND CONES

- The So-Good Ice Cream Company makes Cluster Cones. For packaging, they must cover each cone with paper. If the diameter of the top of each cone is 6 cm and its slant height is 15 cm, what is the area of the paper necessary to cover one cone?
- Looking for lateral area.
- $L = \pi r \ell$
- $L = \pi 3(15) = 141.4\ cm^2$

- *814 #2-32 even, 35-39 all = 21*
- *Extra Credit 817 #2, 6 = +2*

ANSWERS AND QUIZ

- 12.3 Answers
- 12.3 Homework Quiz

12.4 VOLUME OF PRISMS AND CYLINDERS

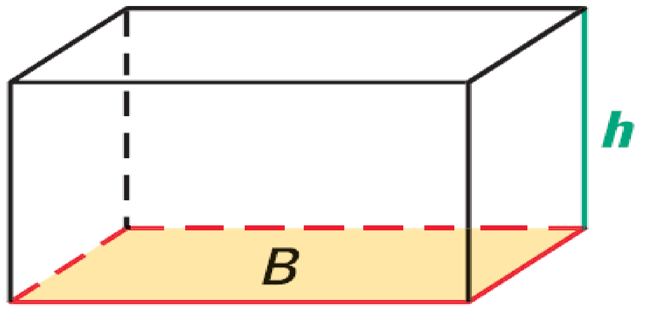
- Create a right prism using geometry cubes
- Count the lengths of the sides
- Count the number of cubes.
- Remember this to verify the formulas we are learning today.

12.4 VOLUME OF PRISMS AND CYLINDERS

Volume of a Prism

$$V = Bh$$

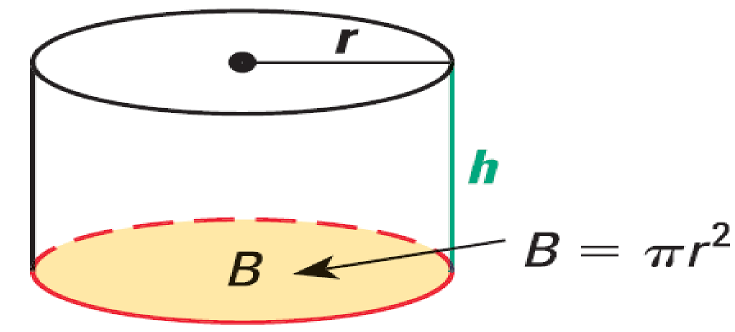
Where B = base area, h = height of prism



Volume of a Cylinder

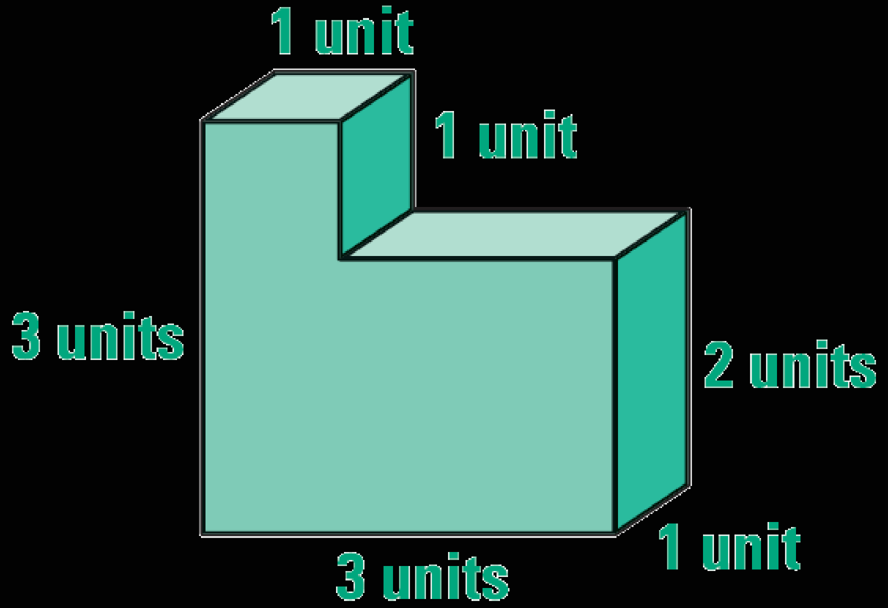
$$V = \pi r^2 h$$

Where r = radius, h = height of cylinder

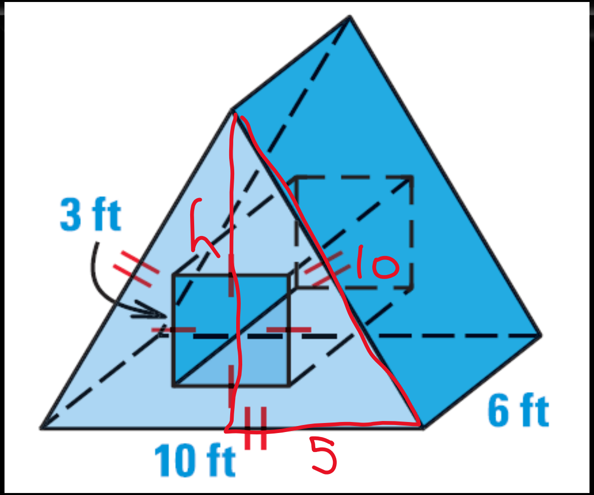


12.4 VOLUME OF PRISMS AND CYLINDERS

- Find the volume of the figure
- Cut into two prisms
- Top cube
  - $V = Bh$
  - $V = 1(1)(1) = 1$
- Bottom
  - $V = 3(1)(2) = 6$
- Total
  - $V = 1 + 6 = 7$



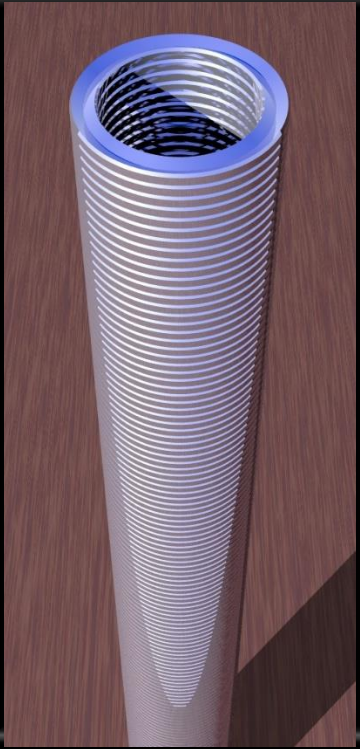
12.4 VOLUME OF PRISMS AND CYLINDERS



- Find the volume.
- Base Area (front)
- Find height of triangle
- $5^2 + h^2 = 10^2$
- $25 + h^2 = 100$
- $h^2 = 75$
- $h = 5\sqrt{3}$

- Base area=triangle - square
  - $B = \frac{1}{2}bh - s^2$
  - $B = \frac{1}{2}(10)\left(5\sqrt{3}\right) - 3^2$
  - $B = 25\sqrt{3} - 9 \approx 34.301$
- $V = Bh$
- $V = \left(25\sqrt{3} - 9\right)(6) \approx 205.8$

12.4 Volume of Prisms and Cylinders



There are 150 1-inch washers in a box. When the washers are stacked, they measure 9 inches in height. If the inside hole of each washer has a diameter of ¾ inch, find the volume of metal in one washer.

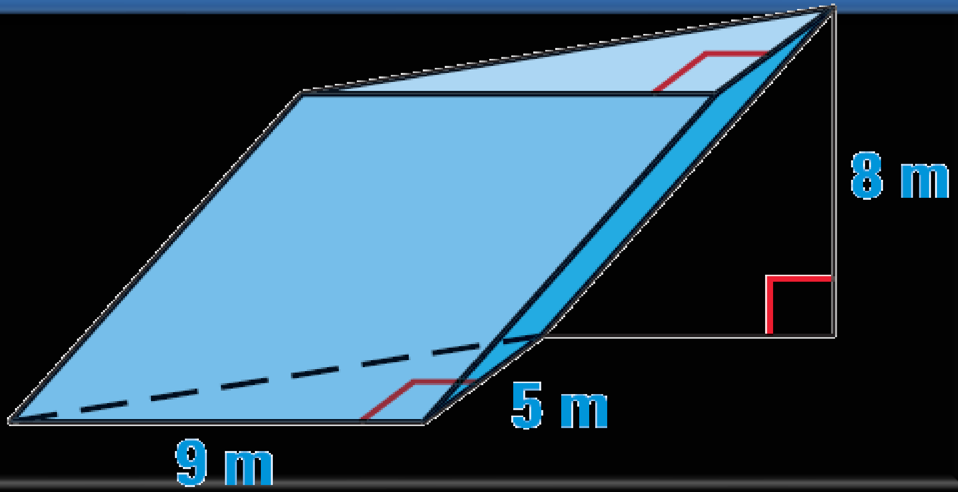
- Base = Big circle – Sm circle
- $B = \pi r_{big}^2 - \pi r_{sm}^2$
- $B = \pi\left(\frac{1}{2}\right)^2 - \pi\left(\frac{3}{8}\right)^2 \approx 0.3436\ in^2$
- Find the height of 1 washer
- $h = \frac{9\ in}{150} = 0.06\ in$
- $V = Bh$
- $V = (0.3436\ in^2)(0.06\ in) = 0.021\ in^3$

12.4 VOLUME OF PRISMS AND CYLINDERS

Cavalieri's Principle

If two solids have the same height and the same cross-sectional area at every level, then they have the same volume.

- Find the volume.
- $B = \frac{1}{2}bh$
- $B = \frac{1}{2}(9)(5) = 22.5\ m^2$
- $V = Bh$
- $V = (22.5\ m^2)(8\ m) = 180\ m^3$



- *822 #2-40 even = 20*

ANSWERS AND QUIZ

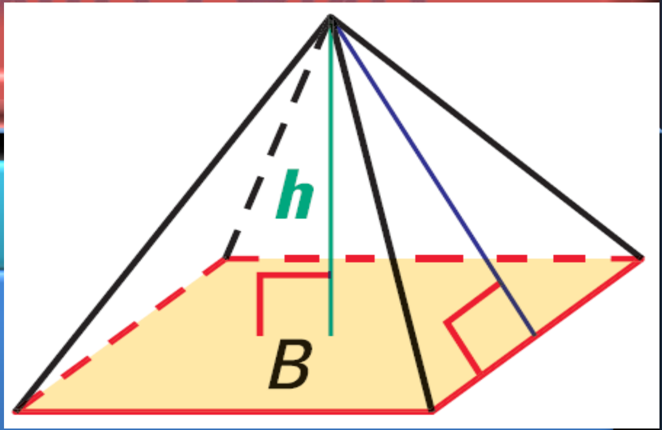
- 12. 4 Answers

- 12.4 Homework Quiz

12.5 VOLUME OF PYRAMIDS AND CONES

- How much ice cream will fill an ice cream cone?
- How could you find out without filling it with ice cream?
- What will you measure?

12.5 VOLUME OF PYRAMIDS AND CONES



Volume of a Pyramid

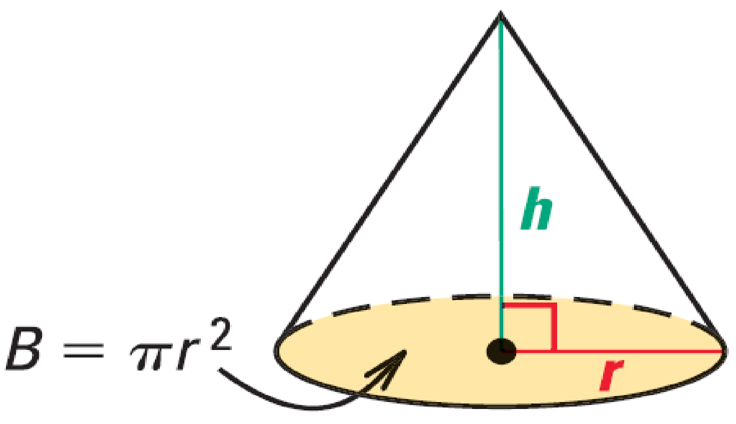
$$V = \frac{1}{3}Bh$$

Where B = base area, h = height of pyramid

Volume of a Cone

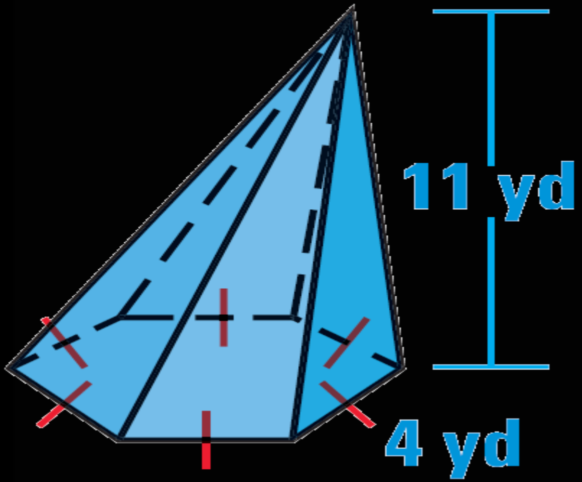
$$V = \frac{1}{3}\pi r^2 h$$

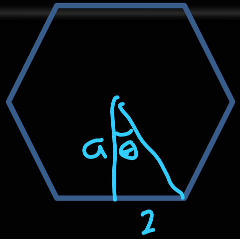
Where r = radius, h = height of cone



12.5 VOLUME OF PYRAMIDS AND CONES

- Find the volume.

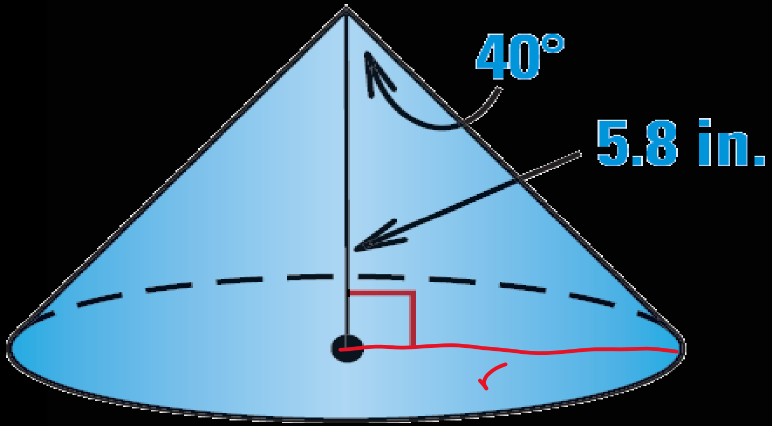




- $B = \frac{1}{2}Pa$
- $\frac{1}{2} central\ \angle = \frac{1}{2}\left(\frac{360°}{6}\right) = 30°$
- $\tan 30° = \frac{2}{a}$
- $a = \frac{2}{\tan 30°} = 3.464$
- $B = \frac{1}{2}(4 \cdot 6)(3.464) = 41.569$
- $V = \frac{1}{3}Bh$
- $V = \frac{1}{3}(41.569)(11) = 152.42$

12.5 VOLUME OF PYRAMIDS AND CONES

- Find the volume.



- *832 #2-30 even, 34, 36, 40, 44-52 even = 23*
- *Extra Credit 836 #2, 4 = +2*

- Find the radius
- $\tan 40° = \frac{r}{5.8}$
- $r = 5.8 \cdot \tan 40° \approx 4.8668$
- $V = \frac{1}{3}\pi r^2 h$
- $V = \frac{1}{3}\pi(4.8668)^2(5.8)$
- $V = 143.86$

ANSWERS AND QUIZ

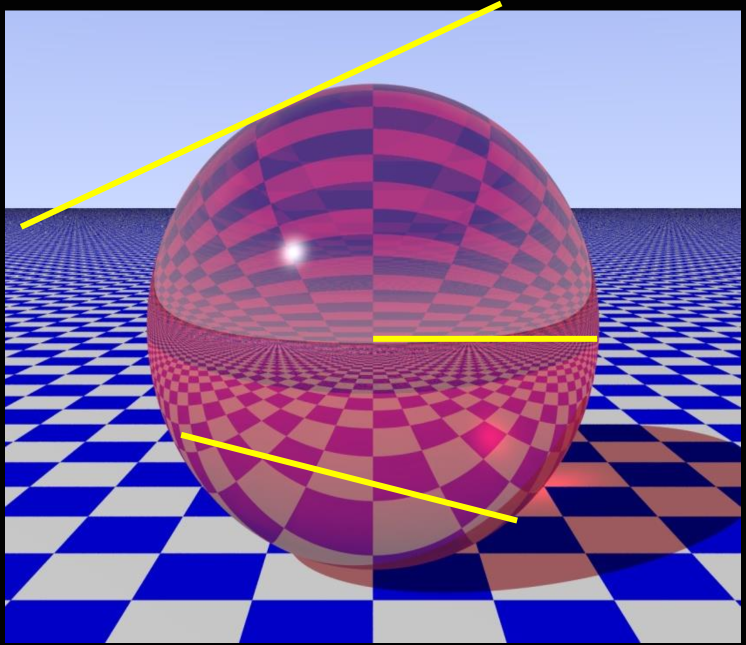
- 12.5 Answers

- 12.5 Homework Quiz

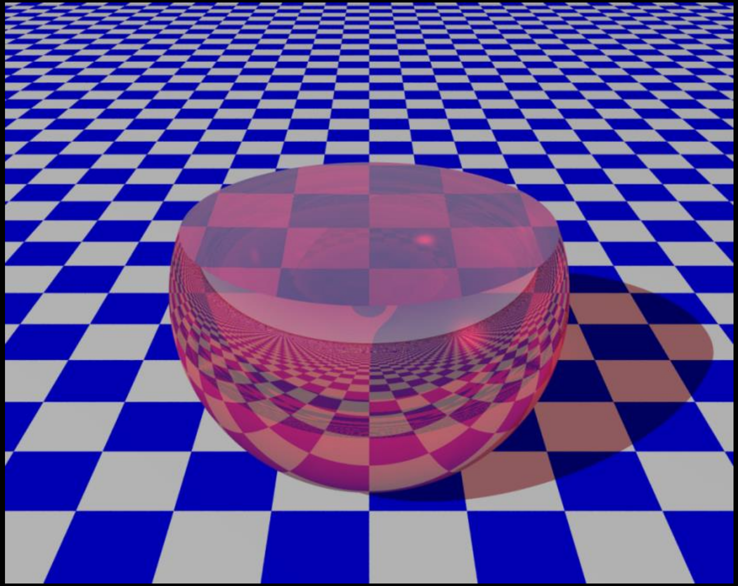
12.6 SURFACE AREA AND VOLUME OF SPHERES

Terms

- **Sphere** → all points equidistant from center
- **Radius** → segment from center to surface
- **Chord** → segment that connects two points on the sphere
- **Diameter** → chord contains the center of the sphere
- **Tangent** → line that intersects the sphere in exactly one place



12.6 SURFACE AREA AND VOLUME OF SPHERES



- Intersections of plane and sphere
  - Point → plane tangent to sphere
  - Circle → plane not tangent to sphere
  - Great Circle → plane goes through center of sphere (like equator)
    - Shortest distance between two points on sphere
    - Cuts sphere into two **hemispheres**

12.6 SURFACE AREA AND VOLUME OF SPHERES

Surface Area of a Sphere

$$S = 4\pi r^2$$

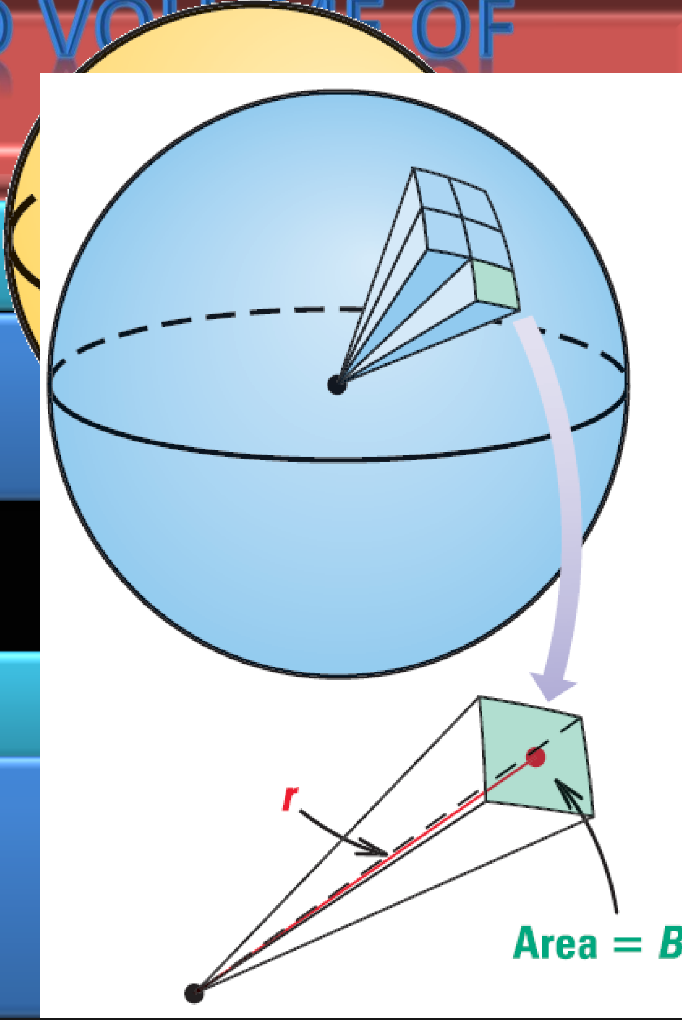
Where r = radius

- If you cut 4 circles into 8ths you can put them together to make a sphere

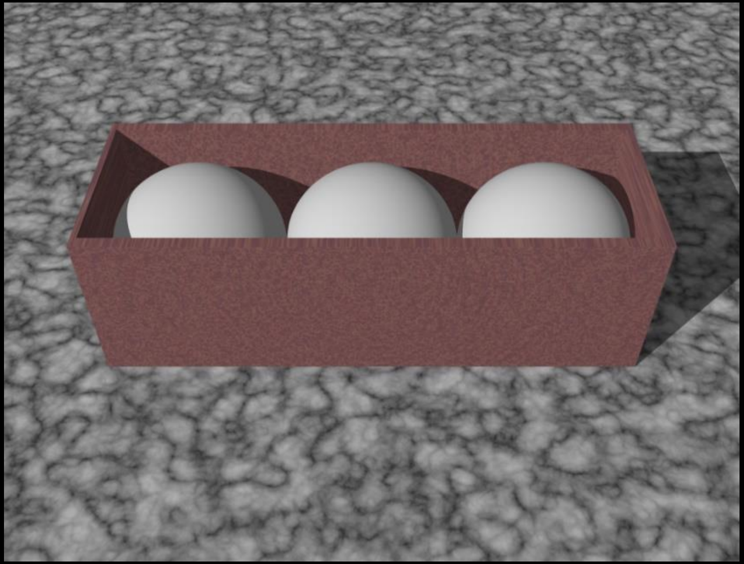
Volume of a Sphere

$$V = \frac{4}{3}\pi r^3$$

Where r = radius



12.6 SURFACE AREA AND VOLUME OF SPHERES



- Find the volume of the empty space in a box containing three golf balls. The diameter of each is about 1.5 inches. The box is 4.5 inches by 1.5 inches by 1.5 inches.
- Volume of box: $4.5(1.5)(1.5) = 10.125$
- Volume of each ball: $V = \frac{4}{3}\pi r^3$
  - $V\frac{4}{3}\pi(0.75)^3 = 1.767$
- Volume of empty space: Box – 3Spheres
- $10.125 - 3(1.767) = 4.824$

- *842 #2-36 even, 40-44 even = 21*

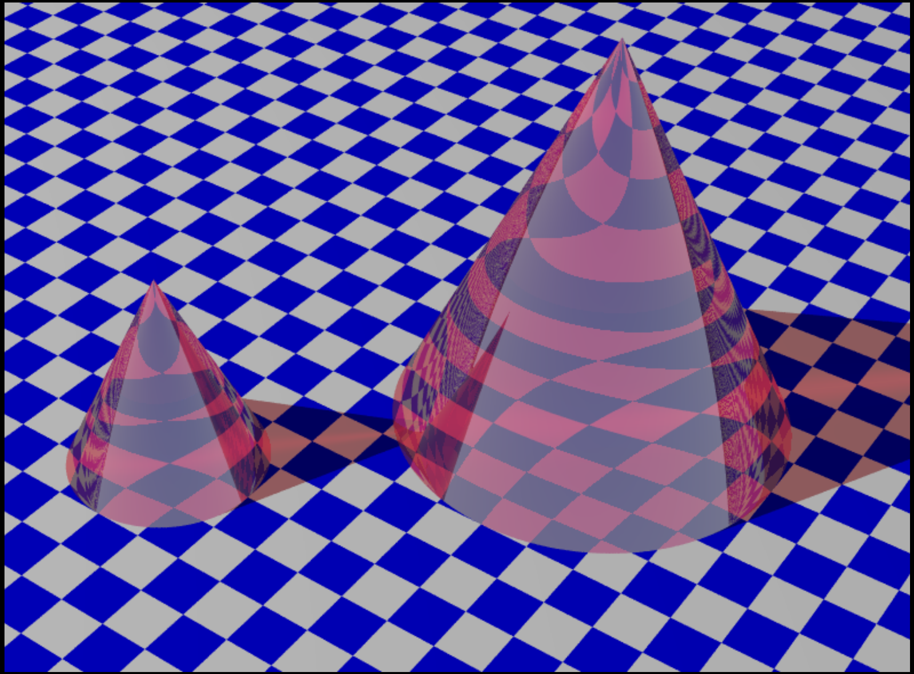
ANSWERS AND QUIZ

- 12.6 Answers
- 12.6 Homework Quiz

12.7 Explore Similar Solids

- Russian Matryoshka dolls nest inside each other. Each doll is the same shape, only smaller. The dolls are similar solids.

12.7 EXPLORE SIMILAR SOLIDS



- Similar Solids
  - Solids with same shape but not necessarily the same size
  - The lengths of sides are proportional
  - The ratios of lengths is called the scale factor

12.7 Explore Similar Solids

- Congruent Solids
  - Similar solids with scale factor of 1:1
- Following four conditions must be true
  - Corresponding angles are congruent
  - Corresponding edges are congruent
  - Areas of corresponding faces are equal
  - The volumes are equal

12.7 EXPLORE SIMILAR SOLIDS

- Determine if the following pair of shapes are similar, congruent or neither.
  - Cone A: r = 4.3, h = 12, slant height = 14.3
  - Cone B: r = 8.6, h = 25, slant height = 26.4
    - Ratios: $\frac{8.6}{4.3} = 2, \frac{25}{12} = 2.08$. Not proportional so neither
  - Right Cylinder A: r = 5.5, height = 7.3
  - Right Cylinder B: r = 5.5, height = 7.3
    - 1:1 ratio so congruent.

12.7 EXPLORE SIMILAR SOLIDS

Similar Solids Theorem

If 2 solids are similar with a scale factor of a:b,
then the areas have a ratio of $a^2:b^2$
and the volumes have a ratio of $a^3:b^3$

12.7 EXPLORE SIMILAR SOLIDS

- Cube C has a surface area of 216 square units and Cube D has a surface area of 600 square units. Find the scale factor of C to D.
- Areas: $\frac{216}{600} = \frac{9}{25} = \frac{c^2}{d^2}$
- Lengths: $\frac{c}{d} = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}$

- Find the edge length of C.
- Cube surface area: $S = 6c^2$
- $216 = 6c^2$
- $36 = c^2$
- $c = 6$

- Use the scale factor to find the volume of D.
- Volumes: $\frac{V_C}{V_D} = \frac{3^3}{5^3}$
- $\frac{6^3}{V_D} = \frac{3^3}{5^3}$
- $\frac{216}{V_D} = \frac{27}{125}$
- $27V_D = 216(125)$
- $V_D = 1000$

- *850 #2-26 even, 30-48 even = 23*
- *Extra Credit 854 #2, 4 = +2*

ANSWERS AND QUIZ

- 12.7 Answers
- 12.7 Homework Quiz